A prior comprehension of the number $l = r\sqrt[n]{\pi^m}$

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Abstract

On this manuscript it shows some algebraic results to understand the origin and meaning of number $l = r\pi^{\frac{m}{n}}$

[1]

1 Introduction

On ref[3] was given the **property 1** about the relation between π, diameter and radius like part of a bounded set where π defines special properties when an iteration sum and square root is applied to such number. Such definition was used to settle two important results about "Squaring the Vector Circle" i.e. the squaring the circle problem analyzed with vector projection. To know the length $l = r\sqrt{\pi}$ that will give place to the equality $A_{Square} = A_{Circle}$ and the relation between the number $\sqrt{\pi} + r$ and the golden number ϕ.

By the other hand; on ref [2] was given a "generalization" of the number $l = r\sqrt{\pi}$ to be used to explain by construction the Poncare´s Conjecture and in ref[1] to explain by induction the "Goldbach´s Conjecture". On such cites was mention some use of such number; but any kind of proof was given to define a more formal argumentation about such numerical definition:

On ref [1] and [2] was mention that ... "Any kind of number *rational* or *irrational* can be written as":

$x = \frac{p\sqrt[n_1]{\pi^{m_1}}}{q\sqrt[n_2]{\pi^{m_2}}}$

was mention to that early definition include prime numbers.

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2 The bounded set of π

Property 1 (ref[3]) establish that

$r < d < \pi$

this relation defines an upper bounded set where π is a *supremum*; hence powering each term by $\frac{m}{n}$ we have:

$r^{\frac{m}{n}} < d^{\frac{m}{n}} < \pi^{\frac{m}{n}}$

and multiplying by r

$r \cdot r^{\frac{m}{n}} < r \cdot d^{\frac{m}{n}} < r \cdot \pi^{\frac{m}{n}}$

By the other hand if we divide every term of last inequality by $r_1 \cdot \pi^{\frac{m_1}{n_1}}$ we have;

$\frac{r \cdot r^{\frac{m}{n}}}{r_1 \cdot \pi^{\frac{m_1}{n_1}}} < \frac{r \cdot d^{\frac{m}{n}}}{r_1 \cdot \pi^{\frac{m_1}{n_1}}} < \frac{r \cdot \pi^{\frac{m}{n}}}{r_1 \cdot \pi^{\frac{m_1}{n_1}}}$

Upper term of early inequality is equal to that used on ref[1 and 2].

3 Some properties about $l = r\pi^{\frac{m}{n}}$ and quadratic equation

Lets establish next integrals about $l = r\pi^{\frac{m}{n}}$

With r, m and n scalars we have;

1) $\int_0^{\pi^{\frac{m}{n}}} rdx = rx|_0^{\pi^{\frac{m}{n}}} = r\pi^{\frac{m}{n}}$

2) $r\int_0^{\pi} x^{\frac{m}{n}} dx = r\frac{x^{(\frac{m}{n}+1)}}{\frac{m}{n}+1}|_0^{\pi} = r\frac{\pi^{(\frac{m+n}{n})}}{\frac{m+n}{n}} = r\frac{n \cdot \pi^{(\frac{m+n}{n})}}{m+n}$

if we take the inverse of last result of integration we have;

$\frac{1}{(r\frac{n \cdot \pi^{\frac{m+n}{n}}}{m+n})} = \frac{(m+n)}{rn\pi^{\frac{m+n}{n}}}$

hence

$\frac{(m+n)}{rn\pi^{\frac{m+n}{n}}} = \frac{m}{rn\pi^{\frac{m+n}{n}}} + \frac{n}{rn\pi^{\frac{m+n}{n}}}$

remember that solution to quadratic equation is

$x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$

Identifying terms we have

$2a = rn\pi^{\frac{m+n}{n}}$

$-b = m$

$\sqrt{b^2 - 4ac} = n$

Hence

$a = \frac{rn\pi^{\frac{m+n}{n}}}{2}$

$b = -m$

$c = \frac{n^2-b^2}{-4a}$

$= \frac{n^2-m^2}{-4(\frac{rn\pi^{\frac{m+n}{n}}}{2})}$

$= -\frac{n^2-m^2}{2(rn\pi^{\frac{m+n}{n}})}$

therefore the quadratic equation related with this term is

$\frac{rn\pi^{(\frac{m+n}{n})}}{2}x^2 - mx - \frac{n^2-m^2}{2rn\pi^{\frac{m+n}{n}}} = 0$

4 Eccentricity and $l = r\pi^{\frac{m}{n}}$

Lets consider the dot product define between two consecutive points (x_1, y_1) and (x_2, y_2); that represent two possible values of a certain function $f(x) \in \Re^2$

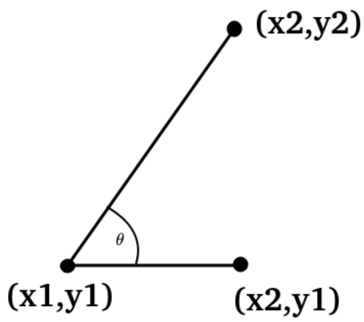

Diagram 1. Vector projection used to define the angle between two consecutive points for a function $f(x) \in \Re$.

$\theta = arccos(\frac{\vec{a}\cdot\vec{b}}{(||\vec{a}||||\vec{b}||)})$

$\theta = arccos(\frac{(x_2,y_2)\cdot(x_2,y_1)}{(\sqrt{x_2^2+y_2^2})(\sqrt{x_2^2+y_1^2})})$

$\theta = (\frac{x_2^2+y_2y_1}{\sqrt{(x_2^2+y_2^2)(x_2^2+y_1^2)}})$

And remember the equation to define eccentricity

$$e = \frac{sin\theta}{sin\alpha}$$

where

$0 < \alpha < 90$

$0 \leq \theta \leq 90$

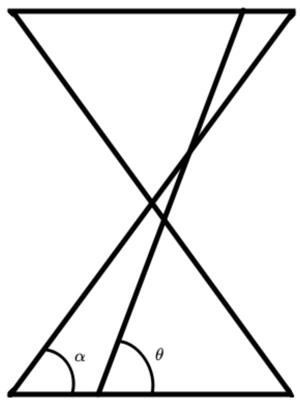

Diagram 2. Double cone used to define conic curves considering its angle.

Considering that sin(x) has like range the closed interval [-1,1]; we could have for the eccentricity; with α constant defining the shape of the double cone (diagram 2)

$\theta = arccos(...)$

with the argument of arccos(...) like was define before.

$sin\theta = r\pi^{\frac{m}{n}}$

to fulfilled the values of range hence

$r = -1$ when $m = 0$ and $r = 1$ when $m = 0$

another case is define for

$r = \frac{sin\theta}{\pi^{\frac{m}{n}}}$

where

$r = \frac{-1}{\pi^{\frac{m}{n}}} = -\pi^{-\frac{m}{n}}$

and

$r = \frac{1}{\pi^{\frac{m}{n}}} = \pi^{-\frac{m}{n}}$

Being valid for any m and $n \in \Re$

In the case of number e we can define

$$e = \frac{r_1 \pi^{\frac{m_1}{n_1}}}{r_2 \pi^{\frac{m_2}{n_2}}} = \frac{sin\theta}{sin\alpha}$$

Early definition is equal to that use on ref[1 and 2] and its expected to be useful on the definition of the properties of conic curves on Goldbach´s Conjecture and Poincare's´ Conjecture.

5 Root $\sqrt{n}$ and Power of π^a

The number $l = r\pi^{\frac{m}{n}}$ makes use of powers of number π

A well definition of its use must require to establish some properties about how to obtain and give values for m and n. A first understanding of $\pi^{\frac{m}{n}}$ distribution on number line could be given by establish next equality;

$\sqrt{n} = l = (1)\pi^{\frac{m}{n}}$

i.e.

$\sqrt{n} = \pi^{\frac{m}{n}}$

next diagram could be of use to define the distribution of $\sqrt{n}$ when $n \in N$; considering the powers of π

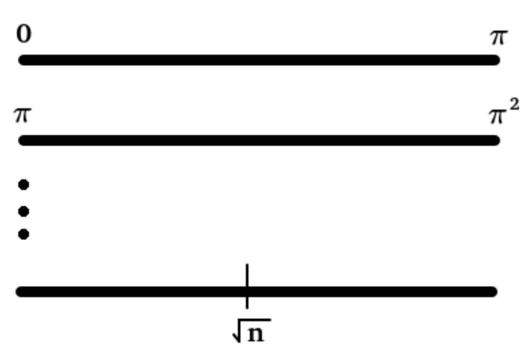

Diagram 3. Scheme of line interval of π powers.

The power a-esim of π will be denoted like π^a. Next table compile main six results for power of π

Inerval $\sqrt{n}$ between π^a	Power π^a	Ratio
$\sqrt{1} \leq \sqrt{n} \leq \sqrt{9}$	$\pi = 3.1416$	-
$\sqrt{10} \leq \sqrt{n} \leq \sqrt{97}$	$\pi^2 = 9.8696$	$\frac{97}{10} = 9.7$
$\sqrt{98} \leq \sqrt{a} \leq \sqrt{961}$	$\pi^3 = 31.0062$	$\frac{961}{98} = 9.8061$
$\sqrt{962} \leq \sqrt{a} \leq \sqrt{9488}$	$\pi^4 = 97.4090$	$\frac{9488}{962} = 9.8627$
$\sqrt{9489} \leq \sqrt{a} \leq \sqrt{93648}$	$\pi^5 = 306.0196$	$\frac{93648}{9489} = 9.8691$
$\sqrt{93649} \leq \sqrt{a} \leq \sqrt{924269}$	$\pi^6 = 961.3891$	$\frac{924269}{93649} = 9.8695$

Table 1: Interval of $\sqrt{n}$ and powers of π

$\sqrt{n}$ distribution between π powers tend to be constant and have a value around 9.869.

Early exercise is useful to define the relation between the magnitude of r and $\pi^{\frac{m}{n}}$; because the power could be used to define certain limits and r in consequence must be less than the values define on each interval of powers. This could be of help to defines a first idea about what kind of functions that needs to be define to establish the properties of r, m and n to establish formally that "$l = r\pi^{\frac{m}{n}}$ could represent any rational or irrational number"

6 References

[1] Alejandro, J. R. (2024). Goldbach´s Conjecture Proof by Induction. J Math Techniques Comput Math, 3(5), 1-3.

[2] Martínez Díaz, J. R. A. (2024). A Topological Construction to Solve Poincare´s Conjecture. Zenodo. https://doi.org/10.5281/zenodo.10965631

[3] Martínez Díaz, J. R. A. (2024). Squaring the Circle and its properties on Hilbert Space. Zenodo. https://doi.org/10.5281/zenodo.11199373